It is, however, possible to know a great deal and understand little. Knowledge without understanding leads to dogmatic methods of work with readily available formulas which, although useful, are insufficient for achieving real successes in the furtherance of science and technology.

In contemporary science, even in a particular narrow special region, it is impossible to know all of the available information, while on the other hand an over-all knowledge is required at the interface of its various branches. Hence it is important that university students obtain a good grasp of universal foundations of knowledge and of methods of problem formulation and solution, and one must strive at imparting to them the maximum deep understanding with minimum information, which is considerable and increases with time. The correct selection of that minimum in the light of prospective developments is a pressing current problem.

Translated by J. J. D.

UDC 539.3

CONTACT PROBLEMS OF THE THEORY OF ELASTICITY IN THE PRESENCE OF WEAR

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The solutions of the contact problems of the theory of elasticity in the presence of wear is given for two cases. In Sect. 1 we consider the problems in which an initially curved beam comes in contact with a half-plane. One of the initial assumptions is that the distance between certain directrices along which the body in contact is sliding and the boundary of the half-plane remains constant. In Sect. 1 the contact between the curved beam and the half-plane is discussed at the assumption that the half-plane is subject to wear. As the result of the wear, the pressure between the beam and the half-plane is gradually reduced. It is naturally assumed that the pressure at the terminal points of the contact area will, in this case, be zero. The conditions characterizing the pressure at these terminal points can be established for various types of contact problems only under certain additional assumptions; this will be discussed below.

1. We assume that the initial form of a fairly thin beam is determined by the initial deflection $w_0(x)$. Let the initial form of the beam be that depicted in Fig. 1a. After the deformation caused by the contact with an rigid half-plane, the beam will assume the form shown in Fig. 1b. We assume that in the present case the beam is symmetric and is in contact with the elastic half-plane along the segment $-a < x < +a$. We shall further assume that the wear is abrasive. In this case the amount of material removed as the result of wear will be proportional to the work done by the frictional forces.

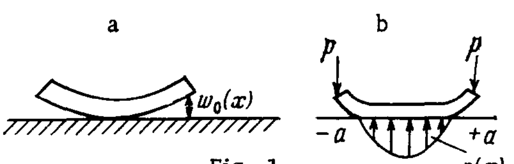

Fig. 1

Experimental results obtained for the abrasive wear can be found in [1, 2]. If the wear is accompanied by a wearing-in process and, in particular, when the bodies in contact are of the same material, the wear may become nonlinear.

Under the above assumptions, the rate of change of the deflection δ will be given by

$$\delta = K^* v\tau = K\sigma, \quad K = K^{**} vK^* \tag{1.1}$$

Here v denotes the averaged modulus of the rate of displacement of the beam (in the direction perpendicular to the plane of the diagram). The shear stress $\tau = K^{**}\sigma$, K^{**} is the coefficient of friction and K^* is the proportionality factor connecting the work done by frictional forces and the amount of material removed.

We have already made the assumption of zero pressure at the edges of the area of contact. This occurs e. g. in the case when the proportionality coefficient connecting the rate of wear with the intensity of the frictional forces is not always constant and vanishes when the pressure becomes fairly small; moreover it is sufficient that these conditions prevail only during the initial stages of the process. A set of experimental data confirms this assumption.

The wear can alter, to some degree, the position of the terminal points of the area of contact. This can however be neglected, since the amount of wear is small compared with the dimensions of the area of contact. The deflection of the beam at the instant t will therefore be

$$w(x, t) = w_0(x) - K\int_0^t p(x, \tau)\, d\tau \tag{1.2}$$

The pressure $p(x, t)$ is connected with the deflection by the following relation:

$$p(x, t) = EI \frac{d^4}{dx^4} w(x, t)$$

Using this we obtain from (1.2)

$$w(x, t) = w_0(x) - KEI \frac{d^4}{dx^4} \int_0^t w(x, \tau)\, d\tau \tag{1.3}$$

We shall seek a particular solution of this equation in the following form:

$$w_\beta(x, t) = e^{-\beta t} w_\beta(x) \tag{1.4}$$

Substituting (1.4) into (1.3), we find

$$(e^{-\beta t} - 1)\, w_\beta(x) = KEI \left(\frac{1}{\beta}(e^{-\beta t} - 1)\right) \frac{d^4 w_\beta}{dx^4}$$

The above equation will hold in the case when $w_\beta(x)$ satisfies the relation

$$\frac{d^4 w_\beta(x^*)}{dx^{*4}} - \left(\frac{\pi}{a}\right)^4 \frac{\beta}{EIK} w_\beta(x^*) = 0, \quad x^* = \frac{\pi}{a} x \tag{1.5}$$

The general solution of this equation is given by

$$w_\beta(x^*) = \lambda_1 \sin \nu x^* + \lambda_2 \cos \nu x^* + \lambda_3 \operatorname{sh} \nu x^* + \lambda_4 \operatorname{ch} \nu x^* \tag{1.6}$$

$$\nu = \sqrt[4]{\frac{\beta}{EIK}}\, \frac{\pi}{a}$$

The eigenvalues and eigenfunctions of Eq. (1.5) can be obtained in the explicit form. We must however use the following boundary conditions for the function $w_\beta(x^*)$ at the points $x^* = -\pi$ and $x^* = +\pi$:

$$w_\beta(x^*) = w_\beta''(x^*) = 0 \text{ for } x^* = \pm\pi \quad \text{(supported ends)} \tag{1.7}$$
$$w_\beta(x^*) = w_\beta'(x^*) = 0 \text{ for } x^* = \pm\pi \quad \text{(clamped ends)}$$

Applying the conditions (1.7) to the function (1.6), we obtain four homogeneous equations

$$\sum_{k=1}^{4} \alpha_{ki}(\nu)\lambda_k = 0 \quad (i = 1, 2, 3, 4)$$

which lead to the following condition:

$$|\alpha_{ki}(\nu)| = 0$$

As the result we obtain a transcendental equation for the eigenvalue ν_n. Each eigenvalue ν_n has a corresponding fundamental function φ_n. These functions can be normed and they form a complete system of orthogonal functions.

Using the second equation of (1.6), we obtain the following expression for β_n

$$\beta_n = \frac{a^4EIK}{\pi^4}\nu_n^4$$

The specified function $w_0(x)$ represents the value of $w(t, x)$ at the instant $t = 0$; it can be expanded into a series in terms of the orthogonal functions φ_n shown above

$$w_0(x^*) = \sum_{n=1}^{\infty} K_n\varphi_n(x)$$

In this case the solution of the problem becomes

$$w(x,t) = \sum_{n=1}^{\infty} K_n \exp\left(-\frac{a^4EIK}{\pi^4}\nu_n^4 t\right)\varphi_n(x^*) \tag{1.8}$$

Investigating the transcendental equations which are used to determine the eigenvalues ν_n, we arrive at the conclusion that these eigenvalues form an increasing sequence. Therefore, in studying the asymptotics at large values of time it is sufficient to restrict ourselves to one or to several terms.

We have thus obtained the exact solution of the problem in question. Below we shall consider the two-dimensional problem which is much more complex. We can also obtain an exact solution for this problem, and it enables us to determine the eigenvalues and eigenfunctions, but in this case they will be for certain integral equations.

We note that the problem considered here resembles the problem of oscillations of a rectilinear rod of constant cross section. A similar method can also be used to solve a more complex problem of wear of a rod which has the form of a nearly circular beam in contact with a rigid circular cylinder.

2. We shall now obtain the solutions of certain two-dimensional contact problems in the presence of wear. We shall consider the action of a die on the boundary of an elastic layer. The die moves relative to this layer in the direction perpendicular to the plane of the diagram shown in Fig. 2. Just as in the cases discussed in Sect. 1, the displacement of the die along certain directrices is such that the distance between these directrices and the elastic layer boundary remains constant. In this case the pressure acting on the layer boundary will, in the presence of wear, gradually diminish.

Assuming that the frictional forces (in the x-direction) are absent, we investigate the contact problems under the following conditions:

1°. The layer rests on a rigid support and there is no friction in the x-direction between the layer and the support.

2°. The layer is rigidly bound to the support.

3°. The modulus of elasticity of the layer material is a function of the coordinate y, and is not zero when $y = 0$). The lower boundary of the layer may lie on the support without friction, or be rigidly bound to it.

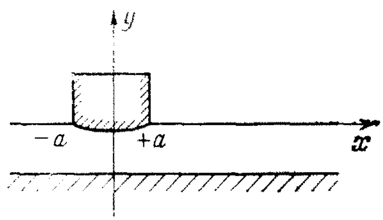

Fig. 2

In all the problems in question the displacement of the surface and the pressure appearing under the die, are connected by the following integral equation

$$w(x) = \int_{-a}^{a} K|x-\xi|\,p(\xi)\,d\xi \qquad (2.1)$$

We note that the above relation has a symmetric kernel. This is due to the fact that the displacement of the boundary caused by the action of the force, is a function of the distance separating the coordinate at which the force is applied from the coordinate of the point of displacement. The simplest known relation of this type is for a half-plane

$$w = A\int_{-a}^{a} \ln|x-\xi|\,p(\xi)\,d\xi$$

It should be noted that the kernels of the type encountered in (2.1) will also have a logarithmic singularity (but not of the higher order). This implies that the kernels of that type will be square integrable, and this feature is essential in the further investigation. Thus we have the kernels

$$K_1|x-\xi|,\ K_2|x-\xi|,\ K_3'|x-\xi| \text{ and } K_3^{('')}|x-\xi|$$

(the last two kernels referring to the third problem where two different conditions may occur at the lower boundary of the elastic layer).

It needs to be said that the problem in which the elastic layer rests on the half-plane without friction is not completely correct, as between the layer and the support a normal displacement may occur. It will be correct under certain specified conditions for a layer possessing weight, as such layer will not detach itself from the support. In this case a somewhat different Green's function will be needed for constructing the integral relation.

We assume that abrasive wear takes place, hence we use the relation (1.1). The arguments given in Sect. 1 concerning the pressure at the edges of the area of contact also remain in force. Using (1.1), we obtain the following equation:

$$w(x,t) = w_0(x) - k\int_0^t p(x,\tau)\,d\tau \qquad (2.2)$$

Here the function $w_0(x)$ describes the area of the die at the initial instant of time, and is known. The expression for $w(x, t)$ can be written in the form of a sum of two terms

$$w(x,t) = w_1(x,t) + w_2(x,t)$$
$$w_1(x,t) = w_{01}(x,t) - k\int_0^t p_1(x,\tau)\,d\tau \qquad (2.3)$$

$$w_2(x,t) = w_{02}(x,t) - k\int_0^t p_2(x,\tau)\,d\tau$$

It is expedient to write the pressure $p\ (x,\ t)$ in the form of a sum of two terms corresponding to the functions $w_1\ (x)$ and $w_2\ (x)$

$$p\ (x,\ t) = p_1\ (x,\ t) + p_2\ (x,\ t)$$

This results from the fact that the pressure $p\ (x)$ at $t = 0$ can be written in the form of a series in terms of the eigenfunctions of a certain homogeneous Fredholm equation. But the eigenvalues for this integral equation can be either positive, or negative. In order to obtain an expression for the pressure which will decrease with time, we must multiply these eigenfunctions by an exponential function of time, and the exponent must contain the time multiplied by a negative coefficient (just as in the expressions (1.4)). Therefore, in order to obtain the pressure and displacements at the successive instants of time, these eigenfunctions must be multiplied by different functions of time.

We solve Eqs. (2.2) and (2.3) using the functions associated with the eigenfunctions of a certain homogeneous Fredholm equation. The functions are constructed using the method given above.

$$\int_{-a}^{a} e^{-\alpha_n'\tau} K\,|\,x-\xi\,|\,p_{n'}(\xi)\,d\xi + \int_{-a}^{a} e^{+\alpha_n''\tau} K\,|\,x-\xi\,|\,p_{n''}(\xi)\,d\xi = \tag{2.4}$$

$$\int_{-a}^{a} K\,|\,x-\xi\,|\,p_{n'}(\xi)\,d\xi + \int_{-a}^{a} K\,|\,x-\xi\,|\,p_{n''}(\xi)\,d\xi -$$

$$K\int_0^t e^{-\alpha_n'\tau}\,p_{n'}(\xi)\,d\tau - k\int_0^t e^{+\alpha_n''\tau}\,p_{n''}(\xi)\,d\tau$$

Equations (2.4) can be written in the form

$$(e^{-\alpha_n' t} - 1)\left\{\int_{-a}^{a} K\,|\,x-\xi\,|\,p_{n'}(\xi)\,d\xi - \frac{K}{\alpha_n'}\,p_1(x)\right\} + \tag{2.5}$$

$$(e^{-\alpha_n'' t} - 1)\left\{\int_{-a}^{a} K\,|\,x-\xi\,|\,p_{n''}(\xi)\,d\xi + \frac{K}{\alpha_n''}\,p_2(x)\right\} = 0$$

Thus, Eq. (2.4) is satisfied if the functions $p_1\ (\xi)$ and $p_2\ (\xi)$ satisfy the following homogeneous Fredholm equation:

$$\int_{-a}^{a} K\,|\,x-\xi\,|\,p(\xi)\,d\xi - \lambda p(x) = 0 \tag{2.6}$$

Here the functions $p_1\ (\xi)$ and $p_2\ (\xi)$ are selected in such a manner, that $p_1\ (\xi)$ can be expanded into a series in the eigenfunctions of the homogeneous Fredholm equation with the corresponding positive eigenvalues, while $p_2\ (\xi)$ can be expanded into a series in the eigenfunctions of the same Fredholm equation with the corresponding negative eigenvalues. This ensures that the system of functions in question is complete.

Equation (2.6) together with the conditions indicated enables us to determine α_n and α_n''. For a positive value of λ_n we have

$$\lambda_n = K / \alpha'_n, \quad \alpha_n' = K / \lambda_n$$

and for a negative value of λ_n we have

$$\lambda_n = -K / \alpha_n''$$

from which it follows that $\alpha_n'' = -K / \lambda_n$.

The initial function $p_0(x)$ which represents the pressure at the initial instant of time, can be expanded into a series in eigenfunctions of the integral equation (2.6) which are orthogonal by virtue of the kernel

$$p_0(x) = \sum_{n=1}^{\infty} A_n p_{n'}(x) + \sum_{m=1}^{\infty} B_m p_{m''}(x)$$

The pressure at subsequent instants of time is given by

$$p(x,t) = \sum_{n=1}^{\infty} A_n \exp\left(-\frac{K}{|\lambda_n'|}t\right) p_{n'}(t) + \sum_{m=1}^{\infty} B_m \exp\left(-\frac{K}{|\lambda_m''|}t\right) p_{m''}(t)$$

where λ_n' denotes the positive, and λ_m'' the negative characteristic numbers.

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UDC 62-50

AN ALTERNATIVE IN THE DIFFERENTIAL-DIFFERENCE GAME OF APPROACH — EVASION WITH A FUNCTIONAL TARGET

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An approach-evasion problem with a functional target set under constraints on the system's trajectory is studied for a conflict-controlled system described by a differential-difference equation. The main result states: either a strategy exists for the first player resolving the approach problem or a strategy exists for the second player resolving the evasion problem. The paper is closely related to [1—6].

1. We examine the system with aftereffect

$$x^{\cdot}(t) = f(t, x_t(s), u, v), \quad t_0 \leqslant t \leqslant \vartheta \tag{1.1}$$
$$u \in P \subset E_{r_1}, \quad v \in Q \subset E_{r_2}$$

Here x is the n-dimensional phase vector; u and v are the controls of the first and second players; P and Q are compacta; the functional $f(t, x_t(s), u, v)$ is defined